Numerical Calculation of the Fields on the Aperture Plane of an Impulse Radiation Antenna

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***Abstract*—A numerical procedure for the calculation of the fields on the aperture plane of an Impulse Radiating Antenna in the frequency domain is presented. The procedure avoids the calculation of the derivative of the inverse of the Jacobi SN function.**

Keywords- Impulse Radiating Antenna; inverse Jacobi SN function

I. INTRODUCTION

The electric and magnetic fields of the main pulse radiated by an Impulse Radiating Antenna ($E_{MP}(x,y,z)$, $H_{MP}(x,y,z)$) can be calculated in the frequency domain by integrating the electric field tangential to the aperture of the reflector $E_A'(x',y')$, weighted by the Green's function, as follows [1]:

$$\vec{H}_{MP}\left(x,y,z\right)=\oint_S \nabla'\left(G\left(r-r'\right)\right)\times\vec{E}_A'(x',y')\,dy'dx'$$
$$\vec{E}_{MP}(x,y,z)=-j\frac{\mu}{\varepsilon\omega}\nabla\times\vec{H}_{MP};\ \ |r-r'|=\sqrt{(x-x')^2+(y-y')^2+(z-F)^2} \tag{1}$$

where $G(r\text{-}r')$ is the Green's function, F is the focal distance of the IRA and the primed variables are the source coordinates on the aperture of the reflector. The origin of the *x, y,* and *z* coordinates is the focal point of the antenna and the *z* axis is perpendicular to the aperture plane of the antenna. Note that we do not make any far-field approximation, as in the approach presented in [2].

A pair of feeders collinear with the *x'* axis produce the following tangential electric field in the aperture [3]:

$$E_A(x',y')=E_{Ax}(x',y')+iE_{Ay}(x',y')=-\frac{1}{K(m)}\left(\frac{\partial u}{\partial x'}-i\frac{\partial u}{\partial y'}\right) \tag{2}$$

where $u(z')=\mathrm{Re}\left\{sn^{-1}\left(\frac{x'+iy}{b_0m^{0.25}},m\right)\right\}$ and sn^{-1} is the inverse of the Jacobi *sn* function, $z'=x'+iy'$, $b_0=D/2$, D is the diameter of the dish, $K(m)$ is the elliptic integral of the first kind, and m is a constant obtained from $Z_c=120\pi K(m)/K(1-m)$; where Z_c is the input impedance of the antenna.

Very few mathematical software tools have implemented the $sn^{-1}(z)$ function (e.g,. Mathematica). In this paper we propose an alternative numerical procedure for the calculation of the derivatives of the sn^{-1} function.

II. METHOD

The derivation starts with the following identity [4]:

$$sn^{-1}\left(\frac{x}{b},\frac{b^2}{a^2}\right)=\int_0^x f(t)dt=\int_0^x\frac{adt}{\sqrt{\left(a^2-t^2\right)\left(b^2-t^2\right)}} \tag{3}$$

from which Equation can be transformed into:

$$E_{Ax}'(x',y')=\frac{-1}{K(m)}\mathrm{Re}\left(\frac{\partial}{\partial x'}\left(\int_0^{z'}f(t)dt\right)\right)\quad E_{Ay}'(x',y')=\frac{1}{K(m)}\mathrm{Re}\left(\frac{\partial}{\partial y'}\left(\int_0^{z'}f(t)dt\right)\right) \tag{4}$$

The integration path on the complex plane is arbitrary, as the function $f(t)$ is analytical.

After some mathematical manipulations and taking into account that on the aperture plane $\|z'\|<D/2$, Eq. yields:

$$E_{Ax}'(x'+iy')=\frac{b_0}{K(m)m^{0.25}}\frac{\cos(\Theta)}{\left(\left(4x'^2y'^2+(b_1^2+x'^2-y'^2)^2\right)\left(4x'^2y'^2+\left(\frac{b_1^{\,2}}{m}+x'^2-y'^2\right)^2\right)\right)^{1/4}}$$
$$E_{Ay}'(x'+iy')=-\frac{b_0}{K(m)m^{0.25}}\frac{\sin(\Theta)}{\left(\left(4x'^2y'^2+(b_1^2+x'^2-y'^2)^2\right)\left(4x'^2y'^2+\left(\frac{b_1^{\,2}}{m}+x'^2-y'^2\right)^2\right)\right)^{1/4}} \tag{5}$$
$$\Theta=0.5Arg\left(\left(b_1^2+(x'+iy')^2\right)\left(\frac{b_1^2}{m}+(x'+iy')^2\right)\right)$$

As an example, Fig. 1 shows the stream lines of the aperture plane fields of a four-arm IRA with D=1 m, F=0.33 m and Z_c=100 Ω, obtained using combination and rotations of the vector field in Equation (5). Fig.2 shows the application of the Equation (5) to the calculation of the radiation pattern obtained using Equation (1) at a distance r=100D, for two signals f= 10 c/D, f=5c/D and amplitudes V=1 Volt.

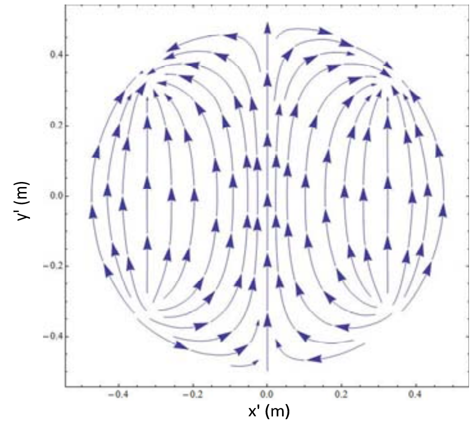

Fig. 1 Fields on the aperture plane of a 4-arm IRA with D = 1 m, Z_c=100 Ω. Notice the arms at and $+-45°$and $+-135°$

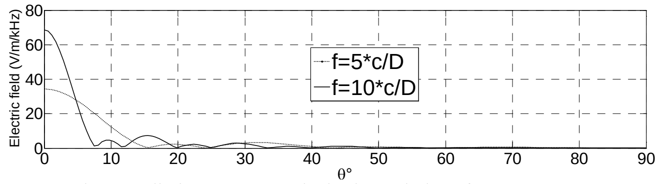

Fig. 2 Radiation pattern on the horizontal plane for a 4-arm IRA

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