Solution of the Fields in a Coaxial Switched Oscillator

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***Abstract*—We present a new design for the electrodes of coaxial switched oscillators using a 3D curvilinear system. The Laplace Equation is solved in the curvilinear space and analytical expressions are derived for the electrostatic field distribution.**

Keywords- Switched Oscillator; Conformal electrodes

I. Coaxial Switched Oscillator

The overall geometry of a coaxial switched oscillator (SWO) is depicted in Figure 1. It is composed of a charged transmission line (coaxial in this case) connected to a higher impedance antenna at one end and to a closing, self-breaking switch gas at the opposite end.

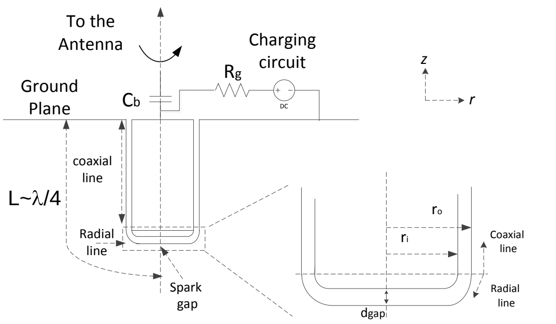

Figure 1. Quarter wave coaxial switched oscillator (SWO). Notice the presence of the electrodes at the bottom end of the coaxial line.

II. Electrodes Profile

We propose to generate the electrodes of the spark gap using two conformal surfaces in a curvilinear orthogonal space, generated from a 2-D transformation called Inverse Prolate Spheroid (IPS), proposed by Moon and Spencer ([1], page 67).

The IPS profile ensures a maximum electric field on the axis of symmetry, and a monotonical decrease when moving towards the coaxial transmission line.

The IPS coordinate (u, v, w) has the following relationship with the Cartesian coordinate system: (1)

$$x = a\frac{\sinh(u)\sin(v)\cos(w)}{\cosh^2(u)-\sin^2(v)} \quad y = a\frac{\sinh(u)\sin(v)\sin(w)}{\cosh^2(u)-\sin^2(v)} \quad z = a\frac{\cosh(u)\cos(v)}{\cosh^2(u)-\sin^2(v)} \qquad (2)$$

where: $a>0$ is a constant and (u,v,w) are defined in the range: $0 \le u < +\infty$; $0 \le v \le \pi$; $0 \le w \le 2\pi$.

A 3D representation of the curvilinear system is shown in Figure 2. It can be seen that the surface u=*constant* forms an inverted prolate spheroid of revolution, while the surface v=*constant* forms an inverted double sheet hyperboloid of revolution and the surface w=*constant* forms a plane.

The electrodes are formed taking two u=*constant* surfaces (u=u_2 and u=u_1) connected to the outer and inner conductors of the coaxial transmission line of the SWO.

Laplace Equation is r-separable in this system. The solution of the electrostatic potential is of the form:

$$V(u,v,w) = \frac{V_1}{k_1}\log\left(k_3\coth\left(\frac{u}{2}\right)\right)\sqrt{\frac{\cosh^2(u)-\sin^2(v)}{k_2^2-\sin^2(v)}} \qquad (3)$$

where k_1, k_2 and k_3 are constants obtained from the boundary conditions of the problem and $V(u_1,v,w)=V_1$ and $V(u_2,v,w)=0$.

Applying the equation for the gradient in the ISP system gives the following expressions for the electric field

$$E_u(u,v,w) = \frac{V_1\sqrt{\Omega}\left(\log\left(\frac{\coth(u/2)}{\coth(u_2/2)}\right)\sinh(2u) - 2\cosh(u)\coth(u) + 2\operatorname{csch}(u)\operatorname{Sin}(v)^2\right)}{ak_1\sqrt{\cosh(2u)-\cos(2v)}\sqrt{\cos(2v)+\cosh(2u_1)}} \qquad (4)$$

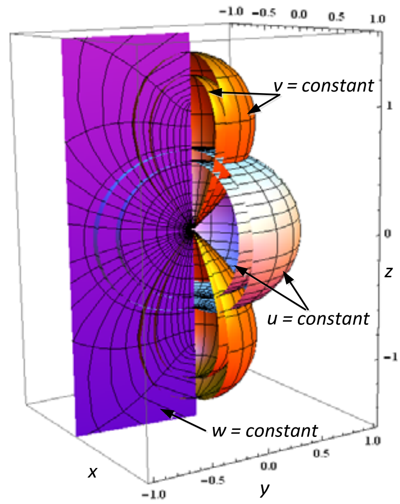

Figure 2. Constant surfaces in the Inverse Prolate Spheroidal coordinate system. The surfaces were generated with parameter a=1. Notice the surfaces corresponding to u-set and v-set.

III. Conclusions

Analytical solution for the electrostatic field distribution in the interelectrodic space of a coaxial SWO was obtained. The derived equation can be used during the design phase of the spark gap. Furthermore, the derived expression allows deriving an analytical solution for the characteristic impedance of the SWO's radial transmission line.

References

[1] P. Moon and D. E. Spencer, Field Theory Handbook: Including Coordinate Systems, Differential Equations and Their Solutions 2ed.: Springer-Verlag, 1971.