ID 067

Calculus of Low Probability-High Consequence Events

Implications for Electromagnetic Hardening Assessment

Edward Toton
Toton LLC
Reston, Virginia 20194 USA
etoton@toton.org

Ira Kohlberg
Kohlberg Associates, Inc.
Reston, Virginia 20190 USA
ira.kohlberg@gmail.com

***Abstract*—We develop a strategy for exploring the relationships between electromagnetic front–and-back door threats, hardening, modeling and testing for systems of all sizes. Real systems can experience hardware and/or logic failure and whose responses may depend critically on design geometry, copy-to-copy variability and stochastic factors. Modeling of the survivability of such systems under such conditions is fraught with risk: a rigorous failure mode analysis cannot be guaranteed to identify all combinations of components and software states that might lead to a critical failure. That is, the *true* probability of survival must always be less than any computed value – and by an unknown amount. We discuss an approach for quantification of survivability bounds that utilizes test results and survived exposures of operational hardware using insights developed by Walley, colorfully illustrated by his "bag of marbles."**

Keywords-survivability; probability; failure; survivability; risk; Bayesian; odds; EMP; hardening

I. INTRODUCTION

This paper extends the notion presented by the authors in a previous AMEREM publication (AMEREM, Ottawa, Canada, July 2010) to the problem of quantification of low probability – high consequence events such as critical system failures from hostile electromagnetic environments. We develop a strategy for exploring the relationships between electromagnetic front–and-back door threats, hardening, modeling and testing for systems of all sizes. Real systems can experience hardware and/or logic failure and whose responses may depend critically on design geometry, copy-to-copy variability and stochastic factors. Modeling of the survivability of such systems under such conditions is fraught with risk: a rigorous failure mode analysis cannot be guaranteed to identify all combinations of components and software states that might lead to a critical failure. That is, the *true* probability of survival must always be less than any computed value – and by an unknown amount. A dramatic example of this type of oversight was encountered in the nuclear meltdown at Three Mile Island, precipitated by the formation of an unpredicted hydrogen bubble forcing coolant away from the reactor core [1]. Since we seek quantification of low-probability failures, these unknown system failure modes become critical to the analysis.

II. APPROACH

We base the strategy for quantification of survivability bounds that utilizes test results and survived exposures of operational hardware on insights developed by Walley, colorfully illustrated by his "bag of marbles" [2]. He asks visitors to his office to guess the probability of drawing a red marble from an opaque bag on his desk; there is no other information about the contents given! The red ball is a clear metaphor for catastrophic system failures. In the absence of information of the nature of such a failure, the Walley questions are: "What is the *highest* probability for a red marble to be drawn for which you are willing to bet that this *will* occur?" and "What is the *lowest* probability for a red marble to be drawn for which you are willing to bet that this *won't* occur?" In the absence of *any* data, these must have the values 0 and 1, respectively. A multivariate analysis with an unknown number of possibilities results in the probability bounds

$$\begin{aligned} P^a &= n_r / (N+s) \\ P^b &= (n_r + s) / (N+s) \\ P^c &= s / (N+s) \end{aligned} \qquad (1)$$

Here, P^a and P^b are the lower and upper bound probabilities for the unknown probability for drawing a red ball or, in our case, encountering an EMP-induced catastrophic system failure, when N draws have been made and n_r red balls drawn. The quantity P^c represents the spread between the lower and upper bounds. The quantity s is a user selected parameter that represents the user's degree of persuasion by the data; Walley suggests that a value of $s = 2$ is reasonable. We see immediately from Eq.(1) that meaningful EMP exposures without system failure, i.e., with $n_r = 0$, have a lower probability bound of 0 and an upper bound of $P^b = P^c = 1/(1 + N/s)$. This shows that repeated survivals forces the upper bound to zero – a reasonable and satisfying quantified behavior.

We have rendered a computational model in [3] based on Walley's concept. This new methodology gives us the opportunity to examine various subsets of threats and tradeoffs between probabilities connected with lethality and survivability, while taking into account the existence of potential unknown EMP-induced catastrophic system failure modes.

REFERENCES

[1] http://www.nrc.gov/reading-rm/doc-collections/fact-sheets/3mile-isle.html

[2] Walley, P. "Inferences from Multinomial Data: Learning about a Bag of Marbles," J. R. Statist. Soc. B (1996) 58. No. 1, pp. 3-57

[3] E. Toton, M. Frankel, and I. Kohlberg, "High Power Electromagnetic Pulse Characterized as Low Probability-High Consequence Events," AMEREM, Ottawa, Canada, July 2010