Statistical Mechanics and Chaos Applied to Electromagnetic Compatibility

Ira Kohlberg
Kohlberg Associates, Inc.,
Reston, Virginia 20190 USA
ira.kohlberg@gmail.com

Robert L. Gardner, Consultant
6152 Manchester Park
Alexandria, Virginia 22310 USA
Robert. L. Gardner@verizon.net

Abstract—A large number of excellent papers have been written in the last 50 years to link the “new” field of chaos to established disciplines of classical mechanics and non-conservative dynamical systems on one side, and set theory and stochastic processes on the other side [1-3]. In this paper we show that many of these chaos-related topics can be folded into a roadmap that demonstrates chaos’s strong dependence on statistical mechanics when applied to electromagnetic compatibility.

Keywords-component: Chaotic dynamical system, stochastic processes, logistic map, butterfly effect, strange attractor

I. BACKGROUND

It’s conjectured that chaos began when Newton’s laws of mechanics started to accurately predict the motion of planetary bodies from mutual gravitational attraction. In retrospect this was relatively easy to do for the two-body interaction because not only was the force between them known but one could also be confident that energy was conserved. The predicted orbits were well defined and essentially periodic. Poincare’s studies of the three-body problem around 1900 showed the existence of non-periodic orbits and bounded in today’s language as a limit cycle connected to a strange attractor. It was also noted that slight differences in initial positions of interacting bodies produced surprisingly huge differences in their orbits —this is one of the cornerstones of chaos theory.

A conclusion from the early studies demonstrated the need for physical models based on statistical mechanics where the focus shifted drastically from a few interacting bodies to gases and fluids. The classic application is Edward Lorenz’s 1961 study of weather prediction. His results showed an acute nonlinear sensitivity to initial conditions. There is also similarity between Lorenz’s equations and those of the Rossler and Chua systems.

II. APPROACH

In this paper we examine the synergism between the Lorenz, Rossler and Chua systems from the instability viewpoint. In particular we seek to better quantify how long these systems can remain in quasi-equilibrium before bifurcation begins. This issue appears to be of high interest in power electronics and power supplies where sub-harmonic bifurcations can connect a single point to a limit cycle [4]. Chaos behavior can appear in a large number of power electronic circuits such as: regulators, current limiter devices, amplifiers, etc.

III. THEORY

A critical feature of the Lorenz, Rossler, and Chua systems is that they are not necessarily energy conserving, and are solved from of the general autonomous equation

$$(d\vec{x}/dt) = \vec{F}(x^{(1)}(t), x^{(2)}(t), \ldots. x^{(N)}(t))$$

By making a connection with turbulence theory Landau and Lifshitz showed that near a limit cycle the foregoing equation can be represented by only two variables—and ultimately modeled by a Poincare map [3]. Ultimately this leads to the solution of the discrete equation

$$x_{j+1} = f(x_j, \lambda) = 1 - \lambda x_j^2$$

In the foregoing equation j is measured in units of the period and λ is a system dependent parameter determined by the bifurcation conditions and the multiplier μ --the numerical factor by which the frequency changes from the stable condition. At this transition point, x_*, it’s required that $x_{j+1} = x_j \equiv x_*$, and $\mu = dx_{j+1}/dx_j$, which then provides the conditions for bifurcation [3]. The foregoing theory, originally applied for turbulence, will be explored to analyze ripple and duty cycle stimulation of sub-harmonics in networks [4]. Related conceptual foundations and limitations of chaos will also be explored.

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