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The Power Wave Theory of Antennas and Some of its Implications

Everett G Farr

Farr Fields, LC
Albuquerque, NM USA
egfarr@gmail.com

Abstract— **We introduce here a simple theory of antenna radiation and scattering that fully extends into the time domain a number of standard antenna terms, including gain, realized gain, antenna pattern, beamwidth, scattering cross section, and radar cross section. Power wave theory applies to linear reciprocal antennas of all feed impedances and feed types, including waveguide feeds. We identify receiving and transmitting impulse responses, and prove that they always have a simple relationship to each other. The approach provides a uniquely useful method of characterizing radiation from and coupling into a complex system, using the same parameters in transmission and reception.**

Keywords-power wave theory; antennas; gain; time domain; impulse response

I. Introduction

We address here the problem of characterizing antenna performance in the time domain. Currently, no standard terms have been defined in the antenna definitions standard [1], which becomes a challenge, for example, when buyers and sellers of wideband antennas need to discuss antenna specifications. In this work, we cast the antenna equations into a particularly simple form. This allows us to very naturally extend into the time domain a collection of commonly used antenna terms, including gain, realized gain, antenna pattern, beamwidth, radar cross section, and scattering cross section. Since complex systems may be considered poor or unintentional antennas, this approach provides a uniquely useful method of describing radiation from and coupling into a complex system. Parts of this paper appeared in [2].

II. Antenna Impulse Response

Let us define far-field antenna performance on boresight for dominant polarization using the parameters defined in Figure 1. This resembles a two-port network, in which Port 2 is a virtual port or radiation port. We define a collection of power waves as

$$\begin{aligned}
\widetilde{\Pi}_{src} &= \frac{\widetilde{V}_{src}}{\sqrt{Z_{o1}}} && = \text{source power wave} \\
\widetilde{\Pi}_{rec} &= \frac{\widetilde{V}_{rec}}{\sqrt{Z_{o1}}} && = \text{received power wave} \\
\widetilde{\Sigma}_{inc} &= \frac{\widetilde{E}_{inc}}{\sqrt{Z_{o2}}} && = \text{incident power flux density wave} \\
\widetilde{Y}_{rad} &= \frac{r\,\widetilde{E}_{rad}}{\sqrt{Z_{o2}}}\, e^{\gamma r} && = \text{radiated radiation intensity wave}
\end{aligned} \quad , \quad (1)$$

where $\gamma = s/v = jk$, $s = j\omega$, $k = \omega/v = 2\pi f/v$ is the propagation constant in the surrounding medium, and v is the velocity of propagation in the medium. Furthermore, Z_{o1} is the real reference impedance of the input port, Z_{o2} is the real impedance of the surrounding medium, and $\widetilde{Z}_{in}$ is the complex impedance looking into the antenna. A tilde indicates a frequency domain quantity. Note that Π, Y, and Σ are Greek versions of P, U, and S, which are commonly used symbols for power, radiation intensity, and power flux density, respectively

With these definitions, we can now define a Generalized Antenna Scattering Matrix (GASM), which is a complete far-field characterization of any linear reciprocal antenna embedded in a lossless medium. First, we treat the special case of dominant polarization on boresight. The antenna equations are expressed compactly as

$$\begin{bmatrix} \widetilde{\Pi}_{rec} \\ \widetilde{Y}_{rad} \end{bmatrix} = \begin{bmatrix} \widetilde{\Gamma} & \widetilde{h} \\ s\,\widetilde{h}/(2\pi v) & \widetilde{\ell} \end{bmatrix} \begin{bmatrix} \widetilde{\Pi}_{src} \\ \widetilde{\Sigma}_{inc} \end{bmatrix}, \quad (2)$$

The proof that these equations are consistent is contained in [5]. In the time domain, this takes the form

$$\begin{bmatrix} \Pi_{rec}(t) \\ Y_{rad}(t) \end{bmatrix} = \begin{bmatrix} \Gamma(t) & h(t) \\ h'(t)/(2\pi v) & \ell(t) \end{bmatrix} \overset{\circ}{\bullet} \begin{bmatrix} \Pi_{src}(t) \\ \Sigma_{inc}(t) \end{bmatrix}, \quad (3)$$

where the "$\overset{\circ}{\bullet}$" operator is a matrix-product convolution operator. Here, $h(t)$ is the antenna impulse response. This can be extended to both polarizations and to arbitrary angles of incidence and observation, as shown in [2].

With this formulation, one can define a transient antenna pattern and beamwidth with respect to a specified norm in the time domain. Mutual coupling can be defined in antenna arrays. Complex systems may be treated as poor antennas, so they may be described with the same parameters.

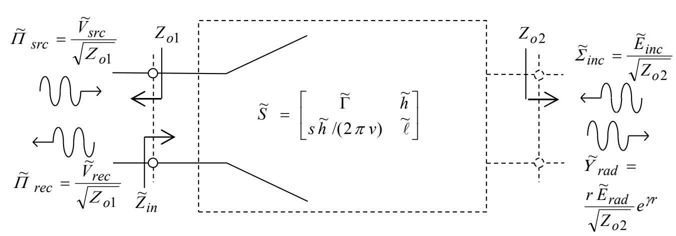

Figure 1. The Generalized Antenna Scattering Matrix (GASM), on boresight, for dominant polarization.

References

[1] IEEE, *IEEE Standard Definitions of Terms for Antennas*, IEEE Std. 145-1993, Institute for Electrical and Electronics Engineering, Inc., New York, March 1993.

[2] E. G. Farr, "A Power Wave Theory of Antennas (With Corrections)," Sensor and Simulation Note 564, May 2013.